

Dimensional Analysis

Math 98 Supplement 2

LEARNING OBJECTIVE

1. Convert one unit of measure to another.

Often measurements are taken using different units. In order for one measurement to be compared to another, it is necessary to convert one unit of measurement to another. For instance, suppose you are visiting Bellingham from Canada. You see the speed limit sign indicating 60mph. How fast should you driving in kilometers per hour? Or, if your height is 165centimeters how tall are in in inches?

“I am 165 cm. How tall am I in inches?”

This problem is an example of unit conversion. You are trying to change one unit (centimeters) to another (inches). To convert a unit, you need conversion equations. These equations tell you how different units are related to one another. For example, these are all examples of conversion equations:

$$\begin{aligned}60 \text{ seconds} &= 1 \text{ minute} \\12 \text{ inches} &= 1 \text{ foot} \\1 \text{ square mile} &= 640 \text{ acres} \\4 \text{ quarts} &= 1 \text{ gallon}\end{aligned}$$

From these equations we get what are called conversion factors.

$$\frac{60 \text{ seconds}}{1 \text{ minute}}, \quad \frac{1 \text{ minute}}{60 \text{ seconds}}, \quad \frac{4 \text{ quarts}}{1 \text{ gallon}}, \quad \frac{1 \text{ gallon}}{4 \text{ quarts}}, \quad \text{etc. ...}$$

Note that these conversion factors (fractions) all equal 1 since the numerator and the denominator of each fraction are equal. You will multiply the quantity with one unit by these conversion factors so that you can eliminate the unwanted units and go to the unit you wish to obtain without changing the original quantity described. (Multiplying a quantity by 1 will not change the original quantity, right? 😊)

Let’s try this problem of converting 165cm to inches using conversion factor!!!

Example 1

165 cm = _____ inches. (Use the conversion equation, 2.54 cm = 1 inch.)

Step 1: Write the quantity you are trying to convert as a fraction. That is, write 165cm as $\frac{165\text{cm}}{1}$.

Step 2: Decide by which conversion factor you need to multiply the above fraction in order to eliminate the “cm” from the numerator. In this problem, the conversion equation relating centimeters and inches is given above. Note that from each conversion equation, you get two possible conversion factors. Your task is to decide which one is the appropriate one to use. In this case you have to choose between

$$\frac{2.54 \text{ cm}}{1 \text{ inch}} \text{ and } \frac{1 \text{ inch}}{2.54 \text{ cm}}$$

Which one should you use and why?

Step 3: Multiply $\frac{165 \text{ cm}}{1}$ by the conversion factor you chose in Step 2. Cancel the unwanted unit and note that you are left with the unit that you needed to obtain. Now simply evaluate the operation as follows.

$$\begin{aligned} \frac{165 \text{ cm}}{1} \cdot \frac{1 \text{ inch}}{2.54 \text{ cm}} &= \frac{165 \cancel{\text{cm}}}{1} \cdot \frac{1 \text{ inch}}{2.54 \cancel{\text{cm}}} = \frac{165 \cdot 1}{1 \cdot 2.54} \text{ inches} \\ &= \frac{165}{2.54} \text{ inches} \approx 64.96 \text{ inches} \end{aligned}$$

So, the final answer is 64.96 inches. **Do not forget to give the correct units with your final answer!**

Try this! (Round your answer to two decimal places if necessary.)

a. 55 kg = _____ pounds
(Use 1 pound = 0.4536 kg)

b. 50 liters = _____ gallons
(Use 1 gallon = 3.7854 liters)

Answer:

a. 125.25 pounds

b. 13.21 gallons

For the next example, let's consider something just a bit more involved.

Example 2

165 cm = _____ feet?

Note that this problem is asking for **feet** in the final answer. So, in addition to the conversion equation we used in Example 1, use 1 foot = 12 inches.

This is an example of a multiple-step problem. In the first example, we converted centimeters to inches in one step.

$$\frac{165 \text{ cm}}{1} \cdot \frac{1 \text{ inch}}{2.54 \text{ cm}}$$

If we stop here; however, your final answer will be in **inches**. This isn't what you want. Now you need to take one more step to go from inches to feet. (**Note! Every time you multiply by a conversion factor, cancel the unwanted unit at each step in order to keep track of where you are in the process.**)

$$\frac{165 \text{ cm}}{1} \cdot \frac{1 \text{ inch}}{2.54 \text{ cm}} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} = \frac{165}{2.54 \cdot 12} \text{ feet} = \frac{165}{30.48} \text{ feet} \approx 5.41 \text{ feet}$$

The final answer is 5.41 feet.

Try this! (Round your answer to the nearest tenth if necessary.)

- a. 55 kg = _____ ounces
(1 pound = 0.4536 kg, 1 pound = 16 oz.)
- b. 250 liters = _____ quarts
(1 gal = 3.7854 liters, 1 gal = 4 quarts)

Answer:

- a. 1940.0 ounces
- b. 52.8 quarts

Some units such as square feet or cubic meters incorporate arithmetical operations. For instance, "cubic meters" means "meters \times meters \times meters". How can we convert these units using conversion factors? To see how this is done, consider the following example.

Example 3

A room is measured to be 115 square meters. What is this area in square feet? Use the conversion equation, 1 meter = 3.28 feet.

Be careful with square or cubic units!!! “115 square meters” means $115 \text{ m}^2 = 115 \text{ m} \cdot \text{m}$. So, as you can see below, using the conversion factor once is not going to eliminate both factors of meters.

$$\frac{115 \text{ m}^2}{1} = \frac{115 \cancel{\text{m}} \cdot \text{m}}{1} \cdot \frac{3.28 \text{ feet}}{1 \cancel{\text{m}}}$$

Note that in the above step, only one of the two factors of meters is canceled. In order to eliminate both of them, you will need to multiply by the conversion factor one more time.

$$\frac{115 \text{ m}^2}{1} = \frac{115 \cancel{\text{m}} \cdot \cancel{\text{m}}}{1} \cdot \frac{3.28 \text{ feet}}{1 \cancel{\text{m}}} \cdot \frac{3.28 \text{ feet}}{1 \cancel{\text{m}}} = \frac{115 \cdot 3.28 \cdot 3.28 \text{ feet} \cdot \text{feet}}{1} \approx 1,237.2 \text{ ft}^2$$

So, 115 square meters is about 1237.2 square feet.

Try this! (Round your answer to two decimal places if necessary.)

a. $150 \text{ sq ft} = \underline{\hspace{2cm}} \text{ sq yards}$
(1 yard = 3 feet)

b. $50000 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ ft}^3$
(1 inch = 2.54 cm, 1 foot = 12 inches)

Careful! This is a multiple-step problem.

Answer:

a. 16.67 square yards

b. 1.77 cubic feet

Now let's consider some examples where the quantities you wish to convert are rates.

The Earth completes its orbit around the Sun roughly in 365 days. If the orbital distance is about 146.6 million kilometers, what is Earth's orbital speed in miles per hour? We will try to do this conversion using conversion factors as follows. First let's consider all the conversion equations we will need to use. We are given that the Earth travels around the Sun 146.6 million km per 365 days. That is, we need to convert

$$\frac{146.6 \text{ million km}}{365 \text{ days}} = \frac{146,600,000 \text{ km}}{365 \text{ days}}$$

into miles per hour.

Example 4

$$\frac{146,600,000 \text{ km}}{365 \text{ days}} = \text{_____ miles/hour}$$

Let's first consider all the conversion equations necessary. Note that for this problem, we need to convert kilometers to miles and days to hours. We will use the following equations:

$$1 \text{ day} = 24 \text{ hours}$$

$$1 \text{ mile} = 1.609 \text{ km}$$

In the original fraction, "km" is in the numerator and "days" is in the denominator. So, we will need the conversion factors,

$$\frac{1 \text{ mile}}{1.609 \text{ km}} \quad \text{and} \quad \frac{1 \text{ day}}{24 \text{ hours}}$$

in order to eliminate these unwanted units. Now we can obtain our answer as follows:

$$\frac{146,600,000 \text{ km}}{365 \text{ days}} \cdot \frac{1 \text{ mile}}{1.609 \text{ km}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} = \frac{146,600,000 \cdot 1 \cdot 1 \text{ miles}}{365 \cdot 1.609 \cdot 24 \text{ hours}} \approx 10,400.97 \text{ mph}$$

So, the Earth is going around the Sun at the speed of 10,400.97 mph. Did you know that we were traveling around the Sun at this high speed???

Try this! (Round your answer to two decimal places if necessary.)

- a. 600 inches/min = _____ ft/sec b. 450 cm/min = _____ mph
(1 in = 2.54 cm, 1 ft = 12 in, 5,280 ft = 1 mile)

Answer:

- a. 0.83 feet per second b. 0.17 miles per hour

TOPIC EXERCISES

(Refer to the list of conversion equations attached at the end of this packet.)

Convert each of the following using conversion factors. Round your answer to two decimal places if necessary.

1. 43 miles = _____feet
2. 857 cm = _____inches
3. 37 mL = _____teaspoon
4. 10 acres = _____hectares
5. 85 kg = _____pounds
6. 2.6 gallons = _____pints
7. 0.2 ton = _____ounces
8. 476 cm = _____yards
9. 1.2 furlong = _____fathom
10. 26 pints = _____L
11. $237 \text{ cm}^2 = \text{_____in}^2$
12. $3500000 \text{ ft}^2 = \text{_____mi}^2$
13. $1290000 \text{ yd}^2 = \text{_____mi}^2$
14. $125 \text{ meter}^3 = \text{_____ft}^3$
15. $3.7 \text{ L} = \text{_____in}^3$
16. $64.52 \text{ ft/sec} = \text{_____mi/hr}$
17. $1.5\text{km/day} = \text{_____meters/min}$
18. $3 \text{ cents/day} = \text{_____dollars/year}$
19. $5 \text{ grams/mL} = \text{_____lbs/gallon}$
20. $2.4 \text{ lbs/qt} = \text{_____kg/L}$

Answer each of the following questions using conversion factors. Round your answer to two decimal places if necessary.

21. A new born baby weighs 2900 grams. How many pounds does this baby weigh?
Also, how many pounds, how many ounces is it?
22. The gas tank of a car holds 65 liters of gas. How many gallons of gas does this tank hold?
23. If a recipe requires $2 \frac{1}{4}$ cups of milk, how much milk is this in mL?
24. A house is built on a 0.27 acre of land. How many square miles is this land?
25. Is it reasonable for a new born human baby to be 1500mm in length? Explain your answer.
26. A river boat requires water depth of 2 fathoms. If the depth of the water is 2.5 meters, can this boat safely operate in this river?
27. One serving of dessert contains 5 teaspoons of sugar. If you need to prepare 250 servings of this dessert for a party, how many cups of sugar do you need?
28. 1 serving of ice cream is $\frac{1}{2}$ cup and contains 285 calories. If you eat 25 teaspoons of this ice cream, how many calories do you consume?

29. You need to rent a moving truck which has a capacity of 750 cubic feet. If a truck has 23 cubic meters of capacity, is this a big enough truck for your move? Explain your answer.
30. A company from U.K. needs to purchase a parcel of land which is at least 2.5 square km. If an U.S. company offers to sell a parcel of land with an area of 35000 square feet, is this big enough for the U.K. company to consider purchasing?
31. A marathon is a race over 42.195 km. If a runner runs this distance in 2 hours, how fast does he run in feet per second?
32. In one city in Florida, it rained 2.2 inches in 50 minutes one afternoon. How many millimeters per hour is this?

ANSWERS

- | | |
|----------------------------|---|
| 1. 227040 feet | 17. 10.4167 m/min |
| 3. 7.4 teaspoons | 19. 41.73 lbs/gal |
| 5. 187.39 pounds | 21. 6.39 pounds \approx 6 lbs 6.29 oz |
| 7. 6400 ounces | 23. 540 mL |
| 9. 132 fathoms | 25. No |
| 11. 36.74 in ² | 27. 26.04 cups |
| 13. 0.42 mi ² | 29. Yes |
| 15. 225.79 in ³ | 31. 19.23 feet/sec |

Conversion Equations:

Length/Distance	
1 m = 100 cm	1 mile = 8 furlong
1 m = 1000 mm	1 mile = 5280 feet
1 km = 1000 m	1 mile = 1.609 km
1 inch = 2.54 cm	1 km = 0.621 mile
12 inches = 1 foot	1 fathom = 6 feet
1 yard = 3 feet	1 nautical mile = 1852 m
1 furlong = 220 yards	1 light year = $5.87849981 \times 10^{12}$ miles
Volume	
1 L (liter) = 1000 cm ³	1 oil barrel = 42 gallons
1 L = 1000 mL	1 teaspoon = 5 mL
1 gallon = 3.7854 L	1 tablespoon = 3 teaspoon
1 pint = 16 fl oz	1 cup = 16 tablespoon
1 quart = 2 pint	1 cup = 240 mL
1 gallon = 4 quart	1 bushel = 32 quart
1 pint = 2 cups	
Area	
640 acres = 1 square mile	1 hectare = 10000 m ²
1 hectare = 2.471 acres	
Mass/Weight	
1 gram = 1000 mg	1 pound = 16 ounces
1 kg = 1000 grams	1 ton = 2000 pounds
1 metric ton = 1000 kg	1 pound = 0.4536 kg
1 kg = 2.2046 pound	