

Ratios, Rates, and Proportions

Math 98 Supplement 1

LEARNING OBJECTIVES

1. Write ratios/rates as fractions in simplest form.
2. Find unit rates.
3. Determine if a proportion is true.
4. Solve proportions.
5. Solve application problems with proportions.
6. Solve problems involving similar figures with proportions.

Ratios and Rates

RATIOS are used, typically, to compare two like quantities. For example, if there are 13 males and 17 females, then the ratio of males to females is 13 to 17.

How to write a ratio.

Ratios can be written in three different ways. In the above example of 13 males to 17 females,

1. In words: 13 to 17.
2. As a fraction: $\frac{13}{17}$.
3. Using a colon: 13 : 17

In all three cases, we read “13 to 17”.

Example 1

During one particular month, the number of sunny days in Bellingham was 4 days while the number of rainy days was 12. Write the ratio of number of sunny days to number of rainy days.

$$4 \text{ to } 12 \quad \text{or} \quad 4:12 \quad \text{or} \quad \frac{4}{12} = \frac{1}{3}$$

As you can see in the above example, since you can write any ratio as a fraction, ratio can be written in simplest form by reducing the fraction.

Example 2

A picture on a wall has length 32 inches and width 12 inches. Write the ratio of length to width in simplest form.

$$\frac{32}{12} = \frac{8}{3}$$

So, the ratio is:

8 to 3 or 8:3 or $\frac{8}{3}$

Now suppose that in example 2, the measurements were given in different units. Consider the following example.

Example 3

A picture on a wall has length 2 feet and width 15 inches. Write the ratio of length to width in simplest form.

Note that in order for us to compare these two measurements and to form a correct ratio, we must have the same units for both quantities. Since 2 feet = 24 inches, length to width ratio is given by

$$\frac{24}{15} = \frac{8}{5}$$

So, the ratio is:

8 to 5 or 8:5 or $\frac{8}{5}$

Try this! Write each ratio in simplest form

a. 42 lbs to 63 lbs

b. 2 weeks to 10 days

Answer:

a. 2 to 3 (or 2:3 or $\frac{2}{3}$)

b. 7 to 5 (or 7:5 or $\frac{7}{5}$)

RATES are used to compare two unlike quantities with different units. For example, if a car is driven for 385 miles on 14 gallons of gas in the tank, then we can describe the comparison of these two quantities as $\frac{385 \text{ miles}}{14 \text{ gallons}}$. Note that the numerator and the denominator have different units. Just as ratios are simplified to lowest terms, so are rates. The rate of $\frac{385 \text{ miles}}{14 \text{ gallons}}$ can be written in simplest form as $\frac{55 \text{ miles}}{2 \text{ gallons}}$ by reducing the fraction. By performing the division, $55 \div 2$, we get what is called a unit rate. In this example since $55 \div 2 = 27.5$ We can describe the same rate as a unit rate of 27.5 miles per gallon. Unit rates are useful in comparing various rates as you will see in an example given below.

Example 4

Write each rate 26 cups for 8 people as a fraction in simplest form, then give the unit rate.

Solution:

$$\frac{26}{8} = \frac{13}{4}$$

So, the rate in simplest form is $\frac{13 \text{ cups}}{4 \text{ people}}$.

Since $26 \div 8 = 13 \div 4 = 3.25$, the unit rate is 3.25 cups per person.

Try this! Write each rate in simplest form, then give the unit rate.

- | | |
|-----------------------------|---|
| a. 30 pencils for 12 people | b. \$27 for 6 lbs of almonds |
| c. 46 hours in 8 weeks | d. 28,770 new jobs created in 60 months |

Answer:

- | | |
|--|---|
| a. $\frac{5 \text{ pencils}}{2 \text{ people}}$, 2.5 pencils per person | b. $\frac{\$9}{2 \text{ lbs}}$, \$4.5 per pound |
| c. $\frac{23 \text{ hours}}{4 \text{ weeks}}$, 5.75 hours per week | d. $\frac{959 \text{ jobs}}{2 \text{ months}}$, 479.5 jobs per month |

Proportions

Two ratios (or two rates) in fractions set equal to each other is called a proportion.

A proportion is true if those two fractions set equal are equivalent. Otherwise, we say that the proportion is false.

Example 5

These are proportions:

a. $\frac{4}{5} = \frac{20}{30}$

b. $\frac{2}{3} = \frac{8}{12}$

c. $\frac{7}{8} = \frac{28}{30}$

Note that of the three proportions above, only b is true since $\frac{2}{3}$ and $\frac{8}{12}$ are equivalent fractions.

How do you know if a proportion is true?

A proportion of the form,

$$\frac{a}{b} = \frac{c}{d}$$

is true if $ad = cb$. In other words, given a proportion, you can check if it is true by cross-multiplication.

To check if $\frac{a}{b} = \frac{c}{d}$ is a true proportion, cross multiply!

$$\begin{array}{ccc} a & = & c \\ \hline b & & d \end{array}$$

$b \cdot c$ $a \cdot d$

Is $ad = bc$? If yes, then it is a true proportion!

Example 6

Are these true proportions?

a. $\frac{5}{7} = \frac{35}{49}$

b. $\frac{13}{15} = \frac{52}{60}$

c. $\frac{6}{7} = \frac{35}{42}$

a. True since $5 \cdot 49 = 7 \cdot 35 = 245$

b. True since $13 \cdot 60 = 15 \cdot 52 = 780$

c. False since $6 \cdot 42 = 252 \neq 245 = 7 \cdot 35$

Try this! Determine if each of the following is a true proportion.

a. $\frac{11}{13} = \frac{63}{78}$

b. $\frac{3}{4} = \frac{81}{108}$

c. $\frac{5}{8} = \frac{85}{136}$

Answer:

a. False

b. True

c. True

Solving proportions

Solving a proportion means to find an unknown quantity within a proportion. We can do so using cross-multiplication. As an example, let's solve the proportion,

$$\frac{7}{x} = \frac{91}{104}$$

In order for this proportion to be true, we must have $91x = 7 \cdot 104$. So,

$$91x = 728$$

Solving this equation by dividing both sides by 91, we find that $x = 8$.

Example 7

Solve each of the following proportions.

a. $\frac{2}{x} = \frac{5}{12}$

Cross-multiply!

$$2 \cdot 12 = 5x$$

$$24 = 5x$$

$$x = \frac{24}{5} \text{ or } 4.8$$

b. $\frac{x+1}{5} = \frac{1}{4}$

Cross-multiply!

$$4(x + 1) = 5 \cdot 1$$

$$4x + 4 = 5$$

$$4x = 1$$

$$x = \frac{1}{4} \text{ or } 0.25$$

c. $\frac{2}{3} = \frac{4x}{x-3}$

Cross-multiply!

$$2(x - 3) = 12x$$

$$2x - 6 = 12x$$

$$-6 = 10x$$

$$x = -\frac{6}{10} = -\frac{3}{5} \text{ or } -0.6$$

Applications

Proportions can be used to solve many application problems. Consider the following example.

Example 8

The distance between City A and City B is 270 miles. On a certain map, this distance is scaled down to 4.5 inches. If the distance between City B and City C on the same map is 12 inches, what is the actual distance between City B and City C?

Solution:

Let x be the actual distance between City B and City C in miles. Since the ratio of actual distance to the distance on the map should be the same between any two cities, we can set up a proportion as follows:

$$\frac{270 \text{ miles}}{4.5 \text{ inches}} = \frac{x \text{ miles}}{12 \text{ inches}}$$

By cross-multiplying, we get:

$$\begin{aligned} 4.5x &= 270 \cdot 12 \\ 4.5x &= 3240 \end{aligned}$$

So... $x = 720$. The actual distance between City B and City C is 720 miles.

Try this!

Suppose you want to enlarge a 4-inch by 6-inch photograph to a poster. If you want the shorter edge of the enlarged poster to be 26 inches, how long is the longer edge of the poster?

Answer: 39 inches

Similar figures

Similar figures are figures whose corresponding sides are proportional. What does this mean?

For example,

Figure 1

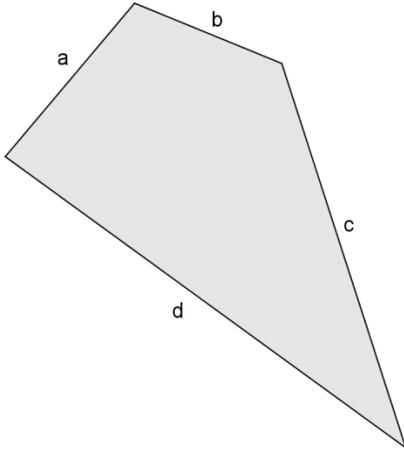


Figure 2

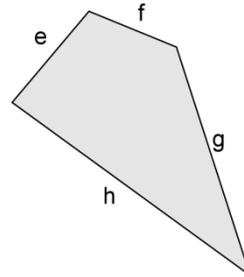


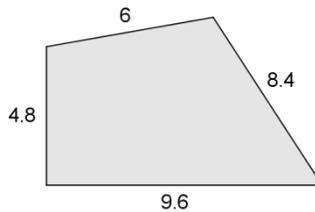
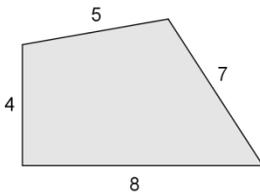
Figure 1 and Figure 2 are similar figures if the corresponding sides are proportional. That is, these are similar if the following relationship is true.

$$\frac{a}{e} = \frac{b}{f} = \frac{c}{g} = \frac{d}{h}$$

If any one of the fraction fails to equal the others, then they are not similar.

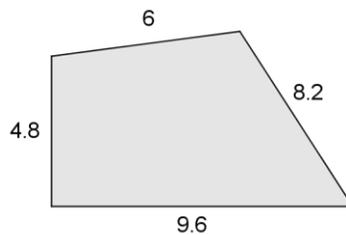
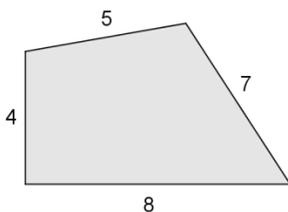
In short...

Similar figures



$$\frac{4.8}{4} = \frac{6}{5} = \frac{8.4}{7} = \frac{9.6}{8} = 1.2$$

Not similar figures

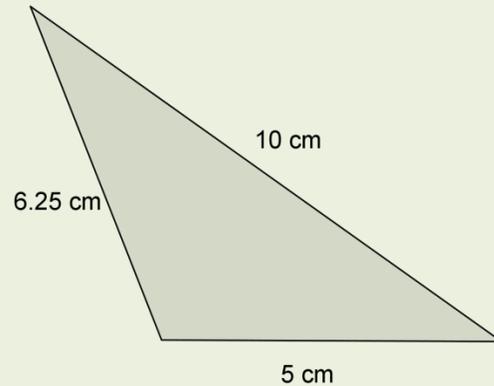
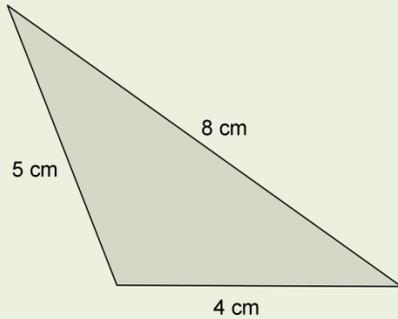


$$\frac{4.8}{4} = \frac{6}{5} = \frac{9.6}{8} = 1.2 \neq \frac{8.2}{7}$$

Example 9

Are these similar figures?

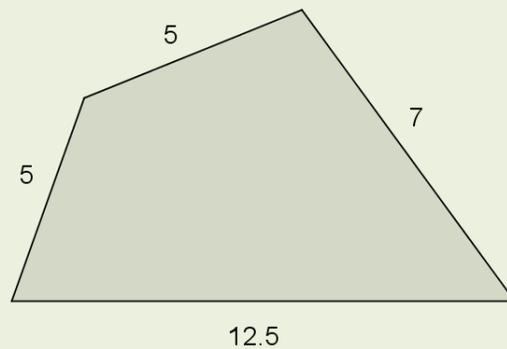
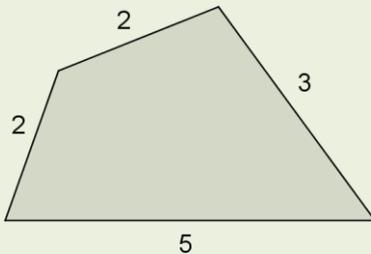
1.



Note that the ratio of the bases of these triangles is $\frac{4}{5}$. If the other two pairs have the same ratio, then these are similar figures. We see that $\frac{4}{5} = \frac{8}{10}$ (how do you know this? Check!) And $\frac{4}{5} = \frac{5}{6.25}$ (check this too!). So... $\frac{4}{5} = \frac{8}{10} = \frac{5}{6.25}$.

Since corresponding sides are proportional **these are similar!**

2.

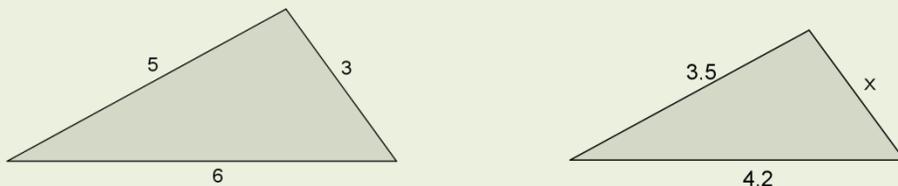


Note that one of the corresponding pair (left side) has the ratio $\frac{2}{5}$. And we also have $\frac{2}{5} = \frac{5}{12.5}$ (check!) However, $\frac{2}{5} \neq \frac{3}{7}$ (check!).

So, **these two figures are not similar.**

Example 10

Given that following figures are similar, find x .



Since corresponding sides are proportional we can write:

$$\frac{x}{3} = \frac{4.2}{6} \quad \text{or} \quad \frac{x}{3} = \frac{3.5}{5}$$

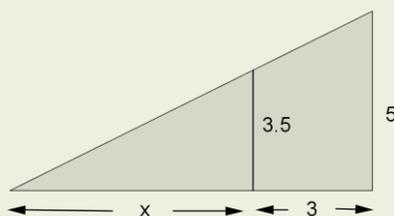
Let's solve the first one to find x . As we learned earlier, we can cross multiply to get

$$\begin{aligned} 6x &= 4.2 \cdot 3 \\ 6x &= 12.6 \\ x &= 2.1 \end{aligned}$$

How about this problem?

Example 11

Find x .



Do you see similar triangles in this diagram? There are two triangles; small one within the larger one. Do you see them? The smaller one's base is x while the base of the larger one is $x + 3$. The height of the smaller triangle is 3.5 whereas the height of the larger one is 5. We can use these two similar triangles to set up a proportion as follows.

$$\frac{x}{x + 3} = \frac{3.5}{5}$$

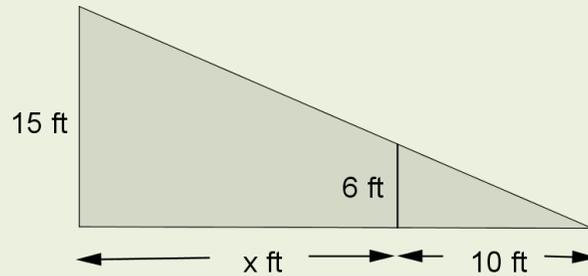
By cross-multiplying you will get $5x = 3.5(x + 3)$. By solving this equation you will see that $x = 7$.

Now let's do some applications.

Example 12

A man 6 feet tall is standing near a street lamp that is 15 feet tall. If his shadow casted on the ground is 10 feet long, how far away is this man standing from the lamp?

You can use similar figures to solve this problem. First, let's draw a diagram.



In this diagram, the street lamp is being represented by the height of the larger triangle, the height of the smaller triangle represents the 6-foot tall man. x represents the distance between the man and the street lamp. Using this diagram, we can set up a proportion like we did in **Example 11** as follows.

$$\frac{6}{15} = \frac{10}{x + 10}$$

By cross-multiplying, we get $6(x + 10) = 10 \cdot 15$. By solving this equation, we see that $x = 15$. So the man is standing 15 feet from the street lamp.

TOPIC EXERCISES

Write each of the following ratios in lowest terms in 3 different ways.

- 18 males to 45 females
- 91 children to 52 adults
- 1 pound to 12 ounces
- 2 feet to 28 inches
- 4 yards to 15 feet
- 2 minutes to 36 seconds

Write each of the following rates in lowest terms and give the unit rate.

- Type 230 words in 4 minutes.
- Earned \$94 in 8 hours.
- 26 lbs of salt in 50 gal of water.
- Increased 5 percent in 20 years.

Check if each of the following proportions is true.

$$11. \frac{51}{136} = \frac{3}{8}$$

$$13. \frac{5}{9} = \frac{85}{171}$$

$$12. \frac{2}{7} = \frac{38}{126}$$

$$14. \frac{52}{91} = \frac{4}{7}$$

Solve each of the following proportions. Round your answer to two decimal places if necessary.

$$15. \frac{7}{5} = \frac{x}{3}$$

$$20. \frac{6}{t-1} = \frac{9}{7}$$

$$16. \frac{4}{3} = \frac{8}{x}$$

$$21. \frac{n}{3-n} = \frac{2}{3}$$

$$17. \frac{9}{6} = \frac{x}{10}$$

$$22. \frac{4-b}{b} = \frac{9}{10}$$

$$18. \frac{10}{8} = \frac{x}{10}$$

$$23. \frac{a}{6} = \frac{8}{3a}$$

$$19. \frac{7}{n+5} = \frac{6}{5}$$

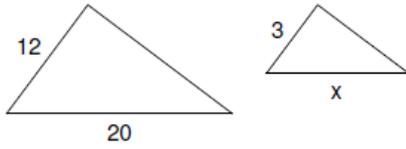
$$24. \frac{5x}{18} = \frac{10}{x}$$

Using proportions, solve the following. Round your answer to the nearest tenth if necessary.

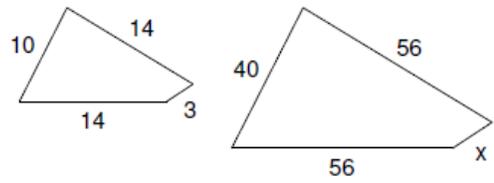
25. Carol spends 17 hours in a 2-week period practicing her culinary skills. How many hours does she practice in 5 weeks?
26. In the year 2000, there were 8.7 deaths per 1000 residents in the United States. If there were 281,421,906 residents in the U.S. during 2000, how many people died that year?
27. Find the distance between Riverside and Victoria if they are 9cm apart on a map that has a scale of 2cm : 39km.
28. Find the distance between Riverside and Fairview if they are 4 in apart on a map that has a scale of 4 in : 52 mi.
29. A piece of cable 8.5 cm long weighs 52 grams. What will a 10-cm length of the same cable weigh?
30. A snowstorm dumped 18 inches of snow in a 12-hour period. How many inches were falling per hour?
31. Mary can read 22 pages in 30 minutes. How long would it take her to read a 100 page book? Write your answer in hours and minutes and round to the nearest minute, if needed.
32. The chance of a woman getting breast cancer in her lifetime is 1 out of 8. At this rate, how many women in a classroom of 32 women would be expected to come down with breast cancer in her lifetime?
33. A company's quality control department found an average of 5 defective models for every 1000 models that were checked. If the company produced 65,800 models in a year, how many of them would be expected to be defective?
34. The ratio of men to women at a private college is 6 to 5. How many women students are there if there are 5592 men?
35. To determine the number of deer in a forest, a forest ranger tags 280 and releases them back into the forest. Later, 405 deer are caught, out of which 45 of them are tagged. Estimate how many deer are in the forest.
36. A worker can complete the assembly of 15 cell phones in 6 hours. At this rate, how many can the worker complete in a 40-hour work week?
37. You find that your watch gains 2 minutes in 5 hours. How much will it gain in 3 days?

38. A yard of fabric costs \$12.99. How much will 2 feet cost?
 Given that each of the following pairs of figures are similar, find x .

39.

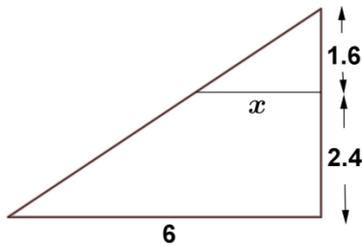


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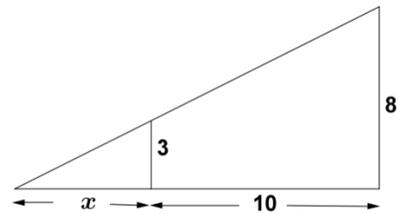


Find x . Round your answer to the nearest tenth if necessary.

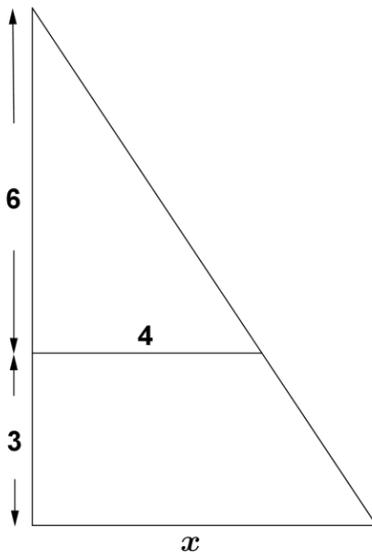
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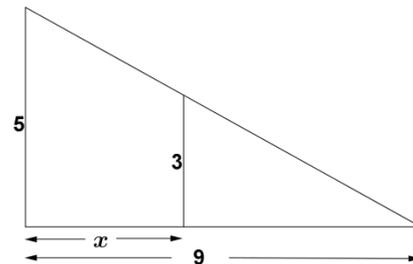
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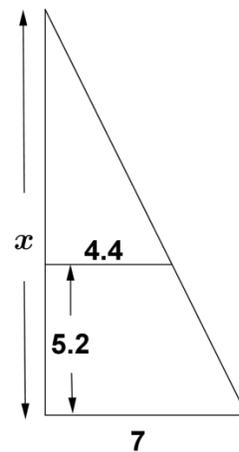
42.



44.



45.



Draw a picture then set up a proportion to solve each of the following problems.

46. A 40 ft flagpole casts a 25 ft shadow. Find the length of the shadow cast by a nearby building that is 200 feet.
47. A 6 ft tall man standing next to a tree casts a 8 ft shadow. If the tree is 15 ft tall, how long is its shadow?
48. A tower casts a shadow 7m long. A vertical stick casts a shadow 0.6 m long. If the stick is 1.2 m high, how tall is the tower?
49. A 1 m tall globe standing next to a baby giraffe casts a 1.5 m shadow. If the baby giraffe casts a shadow that is 6 m long then how tall is it?
50. A girl 160 cm tall stands 360 cm from a lamp post at night. Her shadow from the light is 90 cm long. How tall is the lamp post?
51. A 6 ft tall man is standing 2 feet from a street lamp. If his shadow from the light is 10 ft, how tall is the street lamp?
52. You are standing near a street lamp which is 12 ft tall. Suppose you are 5.5 ft tall and your shadow casted on the ground is 10 ft long. How far away are you standing from the street lamp?
53. Your friend who is 1.8 m tall is standing near a lamp post 8 m tall. If he is standing 19 m from the lamp post, how long is his shadow on the ground? Round your answer to the nearest tenth if necessary.

ANSWERS

1. 2 to 5, 2: 5, $\frac{2}{5}$
3. 4 to 3, 4: 3, $\frac{4}{3}$
5. 4 to 5, 4: 5, $\frac{4}{5}$
7. $\frac{115 \text{ words}}{2 \text{ min}}$, 57.5 words/min
9. $\frac{13 \text{ lbs}}{25 \text{ gallons}}$, .52 lbs/gal
11. True
13. False
15. $x = 4.2$
17. $x = 15$
19. $n \approx .83$
21. $n = 1.2$
23. $a = \pm 4$
25. 42.5 hrs
27. 175.5 km
29. ≈ 61.2 grams
31. ≈ 2 hrs 16 minutes
33. 329 models
35. 2520 deer
37. 28.8 minutes
39. $x = 5$
41. $x = 2.4$
43. $x = 6$
45. $x = 14$
47. 20 feet
49. 4 m
51. 7.2 feet
53. ≈ 5.5 m